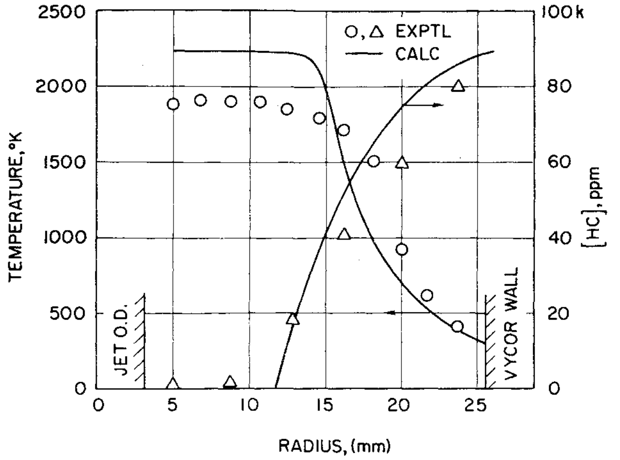

Fig. 4 OJC exhaust HC and temperature profiles.

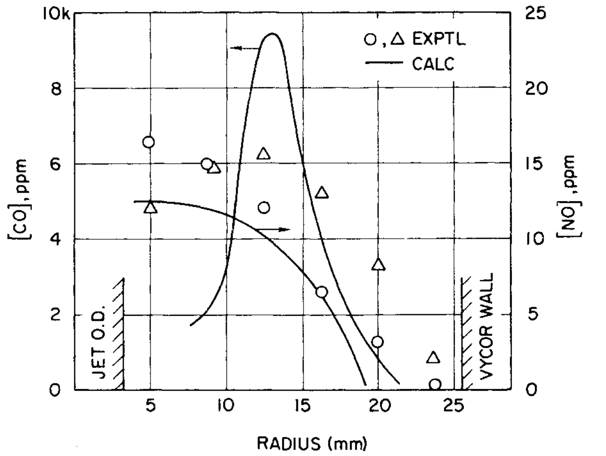

Fig. 5 OJC exhaust CO and NO profiles.

velocity—$V_j = 130$ m/sec; temperature (inlet)—$T_m = T_j = 294$ K; and equivalence ratio—$\phi_m = \phi_j = 1.0$. Predictions of the spatial distribution of flowfield properties are presented in Ref. 3. The predicted location of the high temperature reaction zone begins immediately downstream of the stagnation point and parallels the jet wall to the exit plane with most of the fuel being consumed prior to exhaust. Exhaust plane measurements of temperature and hydrocarbon concentration are compared to predicted values in Fig. 4. The location of the flame front, indicated by sweep temperature and concentration gradients, coincides with theoretical predictions. The temperature deviation may be corrected by considering radiation losses in both the numerical and experimental results.

Fig. 5 presents similar results for CO and NO concentrations. Significant CO concentrations persist throughout the flame zone and reach a maximum in the transition (flame front) region where the temperature and oxygen concentration are relatively low. The indicated trends are favorable, although a sharper peak is predicted for the CO concentration profile than is observed experimentally. This result demonstrates the need to improve the specification of boundary conditions near solid walls as well as the basic chemistry model. The numerically predicted NO profiles indicate that the maximum concentrations appear in the peak temperature zone. The nonequilibrium O-atom calculations yield a 30-fold increase in NO production.

IV. Conclusions

Numerical predictions of the turbulent, backmixed flowfield of a methane-fired opposed-jet combustor have been compared to experimental observations. Although favorable qualitative correlation has been established, a cold flow analysis identified deficiencies in the simplified turbulence model considered and a reacting flow analysis identified deficiencies in the coupled turbulence/kinetic models adopted. The results from this preliminary study provide direction for further experimental verification and formulation of refined kinetic and transport mechanisms.

Acknowledgments

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Parametric Differentiation Technique Applied to a Combustion Problem

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Nomenclature

A_i	= constants of complementary solutions
B	= parameter depending on system considered
$C_i(Y,x)$ $C_i(Z,x)$	= complementary solutions of linear differential equations
D_I	= first Damkohler number
m_I	= concentration of fuel
m_2	= concentration of oxidant
$P(Y,x)$ $P(Z,x)$	= particular solutions of linear differential equations
Pr	= Prandtl number
r	= stoichiometric ratio
Sc	= Schmidt number
T_a	= nondimensional activation energy
T_n	= nondimensional temperature
x	= similarity coordinate
Y	= rate variation of m_2 with respect to a parameter
Z	= rate variation of T_n with respect to a parameter

Subscripts

$\pm\infty$	= values at $\pm\infty$
$'$	= derivative with respect to x

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THE technique of parametric differentiation has been used in many areas by many investigators.[1-7] Goldberg and Richards[1] and Goldberg and Mayers[2] have used this technique for the solution of nonlinear algebraic equations in the analysis of trusses and guyed towers. Kane[3] has solved a set of transcendental equations for real solutions using this technique. Several authors have used this technique for solution of differential equations mainly for compressible boundary-layer flows (Rubberts and Landahl,[4,5] Narayana and Ramamoorthy,[6] and Nath[7]). In all these problems the range of integration is from 0 to ∞ and the gradient of the dependent variable is nonzero at one end of the boundary at least. However, in the problem considered here the range of integration is $-\infty$ to ∞ and gradients are zero at both ends of the boundary. This poses certain problems in solution of linearized equations. In the present Note, the details of the use of the technique of parametric differentiation for a class of problem in the field of combustion with the above mentioned features have been discussed.

Governing Equations and Solutions

Marathe[8] gives a physical basis for the two coupled, ordinary, nonlinear differential equations considered here. The governing equations and boundary conditions are

$$m_2'' + Scxm_2' = D_I Sc \frac{m_I m_2 e^{-T_a/T_n}}{T_n} \tag{1}$$

$$T_n'' + PrxT_n' = -D_I PrB \frac{m_I m_2 e^{-T_a/T_n}}{T_n} \tag{2}$$

where

$$m_I = m_2/r + (m_{I_\infty} + m_{2_{-\infty}}/r)\frac{[1+\mathrm{erf}(x\sqrt{Sc/2})]}{2} - \frac{m_{2-\infty}}{r} \tag{3}$$

with the following boundary conditions

$$x \to -\infty,\ m_2 = m_{2-\infty},\ T_n = T_{n-\infty} \tag{4a}$$

$$x \to \infty,\ m_2 = o,\ T_n = 1 \tag{4b}$$

In this problem any one of the quantities D_I, Pr and Sc can be treated as a parameter. The set of linear and nonlinear equations to be solved for different parameters can be obtained by differentiating the equations and boundary conditions with respect to the parameter considered. With D_I as parameter the equations to be solved are

$$Y'' + ScxY' = Sc\frac{m_I m_2 e^{-T_a/T_n}}{T_n} + D_I Sc \frac{e^{-T_a/T_n}}{T_n}\left[\frac{m_2}{r} + m_I\right] Y + D_I Sc \frac{m_I m_2 e^{-T_a/T_n}}{T_n}\left[\frac{T_a - T_n}{T_n^2}\right] Z \tag{5}$$

$$Z'' + PrxZ' = -PrB\,\frac{m_I m_2 e^{-T_a/T_n}}{T_n} - D_I PrB \frac{e^{-T_a/T_n}}{T_n}\left[\frac{m_2}{r} + m_I\right] Y - D_I PrB \frac{m_I m_2 e^{-T_a/T_n}}{T_n}\left[\frac{T_a - T_n}{T_n^2}\right] Z \tag{6}$$

where

$$Y(x) = \frac{\partial m_2(x)}{\partial D_I} \tag{7}$$

$$Z(x) = \frac{\partial T_n(x)}{\partial D_I} \tag{8}$$

with boundary conditions

$$Y(-\infty) = o,\ Y(\infty) = o \tag{9a}$$

$$Z(-\infty) = 0,\ Z(\infty) = o \tag{9b}$$

Similarly one can write the linear equations when Pr or Sc is chosen as a parameter.

In parametric differentiation technique one solution (exact/true) must be known for a particular set of parameters. Generally this solution is obtained for extreme or limiting values of parameters. In the problem considered, the extreme values of parameters (zero or infinity) give frozen or equilibrium solutions. A special feature concerning these solutions needs mention. The solutions are multivalued for a fixed set of parameters D_I, Pr and Sc (frozen, unstable, and stable branch solution). The nature of the solution is such that one obtains two singularities (implying $\partial T_{n,\max}/\partial D_I \to \infty$) at certain values of D_I which are not known a priori (Fig. 1). Hence we should get solutions separately for the different branches with known solutions on that branch. In order to take account of this feature the initial profiles have been obtained by solving only one differential equation, in the special case of $Pr = Sc$, using the simple bisection method.

Linear Equations

The solution for the set of linear differential equations [Eqs. (5 and 6)] having variable coefficients, is obtained by finding particular and complementary solutions (interpolation method). The variable coefficients can be calculated from the known initial or previous solution. The integration can be started from $-\infty$ or $+\infty$ as at both ends equal number of conditions (two in number) are missing. The main difficulty in starting from either of the ends is that Y' and Z' are so infinitesimally small at these boundaries, that the manipulation of small numbers over reasonable distance coordinate leads to significant errors. Another difficulty is the occurrence of overflows (the computed numbers being very large) during integration. In order to avoid these difficulties, integration of the equations has been performed from $x = 0$ towards both sides with the five sets shown in Table 1.

The first set corresponds to a particular solution and the remaining for complementary solutions. The Runge-Kutta method with Gill modification is used to integrate these equations. Here the values at half intervals are interpolated by third-order Lagrangian interpolation formula. These solutions are combined at $\pm\infty$ to satisfy the boundary conditions (9) as

$$Y(-\infty) = 0 = P(Y, -\infty) + \sum_{i=1}^{4} A_i C_i(Y, -\infty) \tag{10a}$$

$$Y(\infty) = 0 = P(Y, \infty) + \sum_{i=1}^{4} A_i C_i(Y, \infty) \tag{10b}$$

$$Z(-\infty) = 0 = P(Z, -\infty) + \sum_{i=1}^{4} A_i C_i(Z, -\infty) \tag{10c}$$

Table 1 The initial conditions

Set no.	Y	Y'	Z	Z'
1	0	0	0	0
2	1	0	0	0
3	0	1	0	0
4	0	0	1	0
5	0	0	0	1

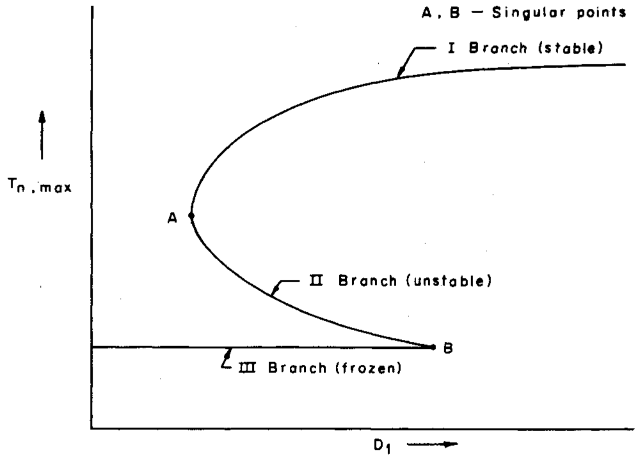

Fig. 1 Solution to the studied problem (schematic).

$$Z(\infty)=0=P(Z,\infty)+\sum_{i=1}^{4}A_iC_i(Z,\infty) \quad (10d)$$

Thus constants of complementary solutions $A_i(i=1, 2, 3, 4)$ can be evaluated from Eq. (10). Knowing these constants and particular and complementary solutions, the profiles of Y, Z, Y' and Z' can be calculated as

$$Y(x)=P(Y,x)+\sum_{i=1}^{4}A_iC_i(Y,x) \quad (11a)$$

$$Z(x)=P(Z,x)+\sum_{i=1}^{4}A_iC_i(Z,x) \quad (11b)$$

$$Y'(x)=P'(Y,x)+\sum_{i=1}^{4}A_iC_i'(Y,x) \quad (11c)$$

$$Z'(x)=P'(Z,x)+\sum_{i=1}^{4}A_iC_i'(Y,x) \quad (11d)$$

Nonlinear Equations

The nonlinear equations [(7) and (8)] are solved for m_2 and T_n at each nodal point (at each nodal value of x). Runge's fourth-order method has been used for integration of these equations. The step size in the parameter is properly chosen so that the resulting solution is independent of step size to the desired accuracy. As Runge's fourth-order method goes through four steps for one increment in parameter step, the linear equations are to be solved four times to get the solution of the next value of the parameter.

Table 2 shows the effect of values of infinities on maximum temperature. Initial profiles (known ones) correspond to the value of D_l equal to 0.1796×10^{14}. Variation in the values of $T_{n,\max}$ is about 0.03% between infinity values taken as ±4 and ±5. The variation is about 0.3% between ±5 and ±6. Theoretically speaking, the variation of $T_{n,\max}$ between ±5 and ±6 should be smaller than that between ±4 and ±5, but because of large roundoff errors due to increased number of calculations, accuracy in computation is lost and we are integrating the equations unnecessarily though the values of Y and Z are small.

Table 2 Effect of values of infinities on maximum temperature[a]

		$T_{n,\max}$		
$x_{-\infty}$	$x_{+\infty}$	$D_l=0.1796E14$[b]	$D_l=0.1814E14$	$D_l=0.1832E14$
−4	+4	5.8379	5.7736	5.7590
−5	+5	–	5.7804	5.7595
−6	+6	–	5.7728	5.7352

[a] $\Delta x=0.05$, $\Delta D_l=0.01$. [b] $0.1796E14=0.1796\times10^{14}$.

To check the accuracy of the solution for the step size in parameter (D_l) and independent variable (x), the usual method of doubling and halving the step size has been used. The computer time needed for the calculations with $x_{\pm\infty}=\pm4$, $\Delta D_l=0.01$ and $\Delta x=0.1$ for one value is about 50 sec on an IBM 360/44.

It can be inferred from the present work that in problems of the kind treated here where the range of integration is $-\infty$ to ∞, the parametric differentiation, a noniterative technique, along with the artifice of integrating from zero on both sides to $-\infty$ and ∞ appears to be a powerful technique.

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Onset of Boiling in Electrohydrodynamic Spraying

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I. Introduction

WHEN a high voltage is applied to a liquid surface, the liquid is often observed to form conical protuberances that emit a spray consisting of ions and charged droplets. This phenomenon, called electrohydrodynamic spraying, has been used commerically in such applications as painting and printing and has been suggested as a source of particles for colloid and ion thrustors for space propulsion. Taylor[1] has pointed out that the liquid cone can exist in mechanical equilibrium, since the increasing electric pressure at the tip is balanced by the increasing surface tension there.

This high electric field implies some sort of electric discharge, supplied by a current flow that heats the cone to a temperature that depends on the thermal and electrical conductivity of the liquid. An earlier paper[2] suggested that this temperature might be high enough to cause the liquid to boil, thus ejecting large particles of liquid, which could ac-

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